PERIODIC SOLUTIONS IN THE PROBLEM OF THREE BODIES*

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1. *Introduction.* In *Les Méthodes Nouvelles de la Mécanique Céleste*, § 48, Poincaré gave a discussion of the periodic solutions of the third kind in the problem of three bodies, which has formed the basis for later researches by Poincaré and other writers. It is the purpose of this note to give an alternative demonstration of the existence of these solutions, using the methods of a preceding paragraph, and certain results of von Zeipel.

2. *Statement of Problem.* Suppose the number of degrees of freedom reduced to four by the methods of § 16,† and the variables of § 18 introduced. Then if

$$\text{(1)} \qquad \begin{cases} x_1 = \Lambda, & x_2 = \Lambda', \quad x_3 = \xi, \quad x_4 = \xi', \\ y_1 = \lambda, & y_2 = \lambda', \quad y_3 = \eta, \quad y_4 = \eta', \end{cases}$$

the equations of motion become,

$$\text{(2)} \qquad \frac{dx_i}{dt} = \frac{\partial F}{\partial y_i}, \qquad \frac{dy_i}{dt} = -\frac{\partial F}{\partial x_i}, \qquad (i = 1, 2, 3, 4).$$

The function F satisfies the relations

$$\text{(3)} \qquad \begin{cases} F = F_0 + \mu F_1 + \cdots, \\ F_0 = F_0(x_1, x_2), \quad \dfrac{\partial^2 F_0}{\partial x_1^2}\dfrac{\partial^2 F_0}{\partial x_2^2} - \left(\dfrac{\partial^2 F_0}{\partial x_1 \partial x_2}\right)^2 \neq 0, \end{cases}$$

in a domain $|x_1 - x_1^0| < b$, $|x_2 - x_2^0| < b$. Also,

$$F_1 = \sum A^{m_1 m_2}_{\alpha_1\alpha_2\beta_1\beta_2} x_3^{\alpha_1} x_4^{\alpha_2} y_3^{\beta_1} y_4^{\beta_2} \cos(m_1 y_1 + m_2 y_2 + h),$$

where $A^{m_1 m_2}_{\alpha_1\alpha_2\beta_1\beta_2}$ is analytic in x_1, x_2 in the domain considered.

We are now in a position to apply the method of § 46. In the notation of that paragraph, the coordinates of a periodic

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† Paragraph numbers refer to the paragraphs of Poincaré's *Les Méthodes Nouvelles*, vol. I.