where D_1 and D_2 are the characteristic functions (2) of the two systems and $p_1 = p(a)/p(b)$. A sufficient condition, then, that the characteristic numbers of the two systems shall either alternate or coincide, is that the quadratic form in $u_1(x)$, $u_2(x)$ shall be definite. But the discriminant Δ of the form is

$$\Delta = (p_1 - 1)^2 u_1^2.$$

Consequently the form will be definite if and only if we have $p(a) = p(b)$, which is the well known condition that system II shall be self-adjoint.*

THE UNIVERSITY OF WISCONSIN

NOTE CONCERNING THE ROOTS OF AN EQUATION

BY K. P. WILLIAMS

Professors Carmichael and Mason have published the following theorem.†

All roots of the equation

$$x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n = 0 \tag{1}$$

are, in absolute value, less than

$$\sqrt{1 + |a_1|^2 + |a_2|^2 + \cdots + |a_n|^2}. \tag{2}$$

It is apparent that this limit may be greatly in excess of the actual maximum of the absolute values of the roots. An illustration of this fact is furnished by the equation

$$x^n + x^{n-1} + \cdots + x + 1 = 0. \tag{3}$$

The theorem asserts that $\sqrt{n+1}$ is greater than the absolute value of any root. If n is large this is rather meager and inexact information, since all roots are in absolute value exactly 1, irrespective of the value of n.

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* *Note on the roots of algebraic equations*, this BULLETIN, vol. 21 (1914), p. 21.

† This example is treated by Bôcher by different means in his *Leçons sur les Méthodes de Sturm*, 1917, pp. 83–91.