ALGEBRAIC GUIDES TO TRANSCENDENTAL PROBLEMS.*

BY R. D. CARMICHAEL

1. *Introduction and Historical Sketch.* Up to the present time algebraic guides to transcendental problems have been employed extensively through only a small part of the range of subject matter to which they are so adapted as to yield important characteristic values in suggested theorems and in methods of proving them. This interaction between algebraic and transcendental analysis has attracted greater attention in the theory of integral equations than elsewhere. The relation between the theory of integral and of algebraic equations seems to have been first noticed by Volterra, who pointed out (TORINO ATTI, 1896, pp. 311–323) that a Volterra integral equation of the first kind may be regarded as in a certain sense a limiting form of a system of n linear algebraic equations in n variables as n becomes infinite. It is clear from Volterra's remarks in 1896 that the same is true of the Volterra equation of the second kind, though this fact was not then mentioned explicitly. In 1913 in his *Leçons sur les Équations Intégrales et les Équations Intégro-différentielles,* Volterra brings out in detail (pp. 30–33, 40–52) the connection between the algebraic theory and his equation of the second kind, and less fully (pp. 56 ff.) the connection between the algebraic theory and his equation of the first kind. He indicates (pp. 71 ff.) extensions of the method to systems of integral equations and to equations and systems with multiple integrals, and also (pp. 138 ff.) to the theory of permutable functions. (See also the preface and pp. 33, 102, 117 for remarks on the history of the subject and for references.) We shall set forth the character of the method by a brief indication of the nature of Volterra's treatment of the equation of the second kind.

* Address as retiring Chairman of the Chicago Section of this Society, Toronto, December 27, 1921. Read for the author by Professor Arnold Dresden.