THE MOMENT OF INERTIA IN THE PROBLEM OF N BODIES*

BY PROFESSOR W. D. MACMILLAN

The differential equations of the problem of n bodies are

$$m_i x_i'' = \frac{\partial V}{\partial x_i}, \quad m_i y_i'' = \frac{\partial V}{\partial y_i}, \quad m_i z_i'' = \frac{\partial V}{\partial z_i}, \quad (i = 1, \cdots, n)$$

in which the potential function is

$$V = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k^2 \frac{m_i m_j}{r_{ij}}, \quad (j \neq i).$$

The kinetic energy of the system is

$$T = \tfrac{1}{2}\sum_{i=1}^{n} m_i(x_i'^2 + y_i'^2 + z_i'^2).$$

The moment of inertia of the n bodies considered as point masses with respect to the origin is

$$I = \sum_{i=1}^{n} m_i(x_i^2 + y_i^2 + z_i^2).$$

On differentiating this expression for the moment of inertia twice with respect to the time, we find

$$\tfrac{1}{2}I'' = \sum_{i=1}^{n} m_i(x_i x_i'' + y_i y_i'' + z_i z_i'') + \sum_{i=1}^{n} m_i(x_i'^2 + y_i'^2 + z_i'^2).$$

The last term of this expression is twice the kinetic energy of the system, $2T$. The first term of the right member, in virtue of the differential equations of motion, may be written in the form

$$\sum_{i=1}^{n}\left(x_i\frac{\partial V}{\partial x_i} + y_i\frac{\partial V}{\partial y_i} + z_i\frac{\partial V}{\partial z_i}\right).$$

Since V is a homogeneous function of degree -1 of the quantities x_i, y_i, z_i, we have

$$\sum_{i=1}^{n}\left(x_i\frac{\partial V}{\partial x_i} + y_i\frac{\partial V}{\partial y_i} + z_i\frac{\partial V}{\partial z_i}\right) = -V.$$

Consequently the differential equation for the moment of

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