cases being that this value may actually be attained in the latter case for one or more special values of θ.*

Finally, it may be observed that the values of the so-called absolute minima for the cases where area may be passed over four, five, six, $\cdots$ times are respectively $\frac{1}{4}l^2\theta$, $\frac{1}{5}l^2\theta$, $\frac{1}{6}l^2\theta$, $\cdots$. The consideration of these cases, however, on the geometrical side again presents serious difficulties, but tends to the opinion, as in the case of triplication, that *in general* the smallest area that can be swept over by any actual movement of angle θ is $\frac{1}{2}l^2\theta$ rather than any of these smaller values.

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CONVERGENCE OF SEQUENCES OF LINEAR OPERATIONS †

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Let U_n be a sequence of linear continuous operations on the class F of functions f, continuous on the interval (a, b), i.e., suppose that every U satisfies the two conditions:

$$(1) \qquad U(c_1f_1 + c_2f_2) = c_1U(f_1) + c_2U(f_2)$$

for every pair of constants (c_1, c_2) and every pair of functions (f_1, f_2) of the class F;

(2) There exists a constant M depending on U such that if Nf is the maximum value of $|f|$ on (a, b) then

$$|U(f)| \leqq MNf.$$

The greatest lower bound of all possible values M might be called the *modulus* of U.

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* Thus, in case $\theta = \pi$ and triplication is allowed, the corresponding value $\frac{1}{2}l^2\pi$ may be attained as follows: Construct the hypocycloid of three cusps obtained by rolling the circle of radius $\frac{1}{2}l$ within the circle of radius $\frac{3}{2}l$ and let the given segment (of length $2l$) move so as to be always tangent to this curve and yet be everywhere entirely within it. The resulting area swept over as θ passes from 0 to π is entirely triplicated, as is well known, and is equal to the amount above stated, $\frac{1}{2}l^2\pi$. See, for example, F. Gomes Teixeira, *Traité des Courbes Spéciales Remarquables Planes et Gauches*, vol. II, p. 193. (Coïmbre, 1909.)

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