These conditions ensure the validity of the theorems* used in the proof of the existence theorem stated above, and the proof follows exactly as in the original theorem.

UNIVERSITY OF TEXAS,
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ON THE CAUCHY-GOURSAT THEOREM.

BY R. L. BORGER.

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In order to prove his integral theorem, viz: $\int_C f(z)dz = 0$, Cauchy found it necessary to assume not only that the derivative $f'(z)$ existed but also that it was continuous. Later, proofs were given by Goursat and by Moore† in which the mere existence of $f'(z)$ was shown to be sufficient for the truth of the theorem. These were based upon the analysis of the complex variable.

From the standpoint of the real variable many interesting investigations have developed around the Cauchy-Goursat theorem. They have depended upon Green's theorem. Porter,‡ using the Riemann integral, proved that with proper restrictions upon the component functions, U and V, of the complex function, Green's theorem was true, and hence that Cauchy's integral theorem was also true, even when the derivative $f'(z)$ did not exist. Montel,§ by means of the Lebesgue integral, proved Green's theorem under the hypothesis that U_x, V_y, exist, are bounded, and satisfy the equation

$$U_x = V_y,$$

except at most in a set of measure zero. He was then able to prove the integral theorem, and the existence of the derivative $f'(z)$ for a function of the complex variable $f(z)$ in the closed region considered. The existence, then, of the deriva-

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* Cf. *Existence theorems for the general, real, self-adjoint linear system of the second order*, TRANSACTIONS AMER. MATH. SOC., vol. 19 (1918), p. 94.

† TRANSACTIONS AMER. MATH. SOCIETY, vol. 1 (1900).

‡ ANNALS OF MATHEMATICS, (2), vol. 7 (1905–6).

§ ANNALES SCIENTIFIQUES DE L'ECOLE NORMALE SUPÉRIEURE, (3), vol. 24 (1907).