$$\lim_{n=\infty} \frac{\pi}{n} \sum_{\nu=0}^{[n/2]-1} \left(\frac{1}{\sin \frac{2\nu+1}{2n}\pi} - \frac{1}{\frac{2\nu+1}{2n}\pi} \right)$$

$$= \int_0^{\pi/2} \left(\frac{1}{\sin x} - \frac{1}{x} \right) dx = \log \frac{4}{\pi},$$

or

$$\frac{2}{n} \sum_{\nu=0}^{[n/2]-1} \frac{1}{\sin \frac{2\nu+1}{2n}\pi} = \frac{4}{\pi} \sum_{\nu=0}^{[n/2]-1} \frac{1}{2\nu+1} + \frac{2}{\pi} \log \frac{4}{\pi} + o(1).$$

Using the familiar asymptotic formula

$$\sum_{\nu=0}^{m} \frac{1}{2\nu+1} = \frac{1}{2}\log m + \frac{1}{2}C + o(1),$$

where C is Euler's constant, we find

$$\frac{1}{n} \sum_{\nu=0}^{n-1} \frac{1}{\sin \frac{2\nu+1}{2n}\pi} = \frac{2}{\pi} \left(\log n + C + \log \frac{2}{\pi} \right) + o(1).$$

TECHNICAL STAFF,
OFFICE OF THE CHIEF OF ORDNANCE

THE MINIMUM AREA BETWEEN A CURVE AND ITS CAUSTIC.

BY PROFESSOR PAUL R. RIDER.

(Read before the American Mathematical Society April 9, 1920.)

If rays from a given source of light are reflected by a curve, the envelope of the rays after reflection is called the caustic of the curve. It is an interesting problem to find the curve which connects two fixed points and which with its caustic and the rays reflected from the fixed points will enclose a minimum area. Euler* proposed and solved a similar problem

* Euler, *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes* or German translation in Ostwald's *Klassiker der exakten Wissenschaften*, no. 46. See also Todhunter, *Researches in the calculus of variations*, chapter 13.