RECIPROCAL SUBGROUPS OF AN ABELIAN GROUP.

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§ 1. *Introduction.* Every two subgroups of the group G which have the property that the product of their orders is equal to the order of G have been called *reciprocal subgroups* of G.* A group may have subgroups which have no reciprocals. For instance, the tetrahedral group contains subgroups of order 2 but it does not contain any subgroup of order 6. If a group is abelian, each of its subgroups is known to have at least one reciprocal subgroup. A necessary and sufficient condition that every subgroup of G have one and only one reciprocal is that G be cyclic.

Two invariant subgroups of G will be called *corresponding reciprocal subgroups* of G if each of them is simply isomorphic with the quotient group of G with respect to the other. One of the objects of the present paper is to prove that every subgroup of an abelian group has a corresponding reciprocal subgroup. All the subgroups in a complete set of conjugate subgroups under the group of isomorphisms of G, that is, all the subgroups in a set of I-conjugate subgroups of G, must evidently have the same reciprocal subgroups. Hence the theory of corresponding reciprocal subgroups of an abelian group establishes a correspondence between pairs of sets of I-conjugate subgroups. In what follows it will be assumed that G is abelian.

As G is the direct product of its Sylow subgroups when the order of G is not a power of a prime number p and as the number of I-conjugates of a subgroup of such a G is the product of the numbers of the I-conjugates of the Sylow subgroups of this subgroup it will be assumed in what follows that the order of G is of the form p^m and that G has λ_1 invariants which are separately equal to p^{m_1}, λ_2 invariants which are separately equal to $p^{m_2}, \cdots, \lambda_\gamma$ invariants which are separately equal to p^{m_γ}. Hence

$$\lambda_1 m_1 + \lambda_2 m_2 + \cdots + \lambda_\gamma m_\gamma = m.$$

It will be convenient to assume that $m_1 > m_2 > \cdots > m_\gamma$.

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