

they may give some satisfaction to those who have delved in the theory of those transformations, since they make the transformations appear, not as separate chapters of the analysis, but as special applications of formulas now well established in other parts of the theory. It is even possible to make a satisfactory and not over complicated sufficiency proof for a weak minimum, without the use of the Weierstrass notion of a field. For the Weierstrass formula can be proved directly with some ease for the second variation when once it has been seen to hold true for a conjugate system of solutions of Jacobi's equations. It is not strange that the second variation has not been attacked from this standpoint before in spite of the fact that in my recent review of the literature I have found several suggestions which might have instigated one to attempt it. The real reason is, I think, that the advances of Weierstrass and Hilbert were published after 1900 and about the time that Kneser found his envelope theorem. The tendency since then has been to discard the theory of the second variation in favor of the more geometrical theory, but the experiment, so far as I know, has not been completely successful.

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GROUPS GENERATED BY TWO OPERATORS OF ORDER THREE WHOSE PRODUCT IS OF ORDER FOUR.

BY PROFESSOR G. A. MILLER.

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§ 1. *Introduction.*

It is known that the only groups which are completely determined by the orders of two generators and the order of their product are the dihedral groups and the groups of movements of the five regular solids known to the ancients. In all other cases two generators which are not restricted except as regards their orders and the order of their product