Replace $2x$ by $x - y$ and $2y$ by $x + y$.

$$S(x - y) = S(x)C(y) - C(x)S(y).$$

If y is replaced by $-y$,

$$S(x + y) = S(x)C(y) + C(x)S(y).$$

Therefore

$$\begin{aligned} S^2(x + y) - S^2(x - y) &= 4S(x)C(x)S(y)C(y) \\ &= S(2x)S(2y). \end{aligned}$$

Replace $2x$ by $x + y$ and $2y$ by $x - y$. Then

$$S(x + y)S(x - y) = S^2(x) - S^2(y).$$

Substituting the relations found, it follows that

$$C(x + y)C(x - y) = C^2(x) + C^2(y) - 1,$$

that is, the odd component of $F(x)$ satisfies (5) while the even component (except for the factor $F(0)$) satisfies (4).

STATE UNIVERSITY OF IOWA,
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THE EQUATION $ds^2 = dx^2 + dy^2 + dz^2$.

BY PROFESSOR E. T. BELL.

1. THIS equation,* being of geometrical importance, has attracted several writers, including Serret (1847), Darboux (1873, 1887), de Montcheuil (1905), Salkowski (1909), Eisenhart (1911), and Pell (1918). The simple parametric solution of de Montcheuil, which is the starting point of considerable work in differential geometry, was not noticed by Serret or Darboux. It is somewhat remarkable that the latter overlooked this solution, as he himself makes use (Surfaces,

* Full references to earlier writers are given by Eisenhart, *Annals of Math.* (2), vol. 13 (1911), pp. 17–35. Pell's paper will be found ibid. (2), vol. 20, pp. 142–148. The substance of the present note, with the exception of section 8, is from an unpublished A.M. thesis, presented to the University of Washington in 1908, dealing with the general algebraic problems on which solutions of this kind depend. I wish to emphasize that § 8 was written only after I had read Pell's paper.