as a $(2 n-1)$-space tangent to the $n$-dimensional quadratic cone $K_{n}{ }^{\prime}$ of ( $n-2$ )-spaces also represented as $Q_{n}{ }^{\prime}$. While a $P^{\prime}$ of $Q_{n}{ }^{\prime}$ meets $Q_{n}$ in a conic, the two ( $2 n-2$ )spaces, $L^{\prime}$, tangent to $K_{n}{ }^{\prime}$ and contained in $P^{\prime}$, which are also tangent to $Q_{n}$, determine two points of tangency on $Q_{n}$. This correspondence is again two-two, and for it the same theorem holds. The case $n=2$ leads to the study of Kummer's surface and the theorem is in substance familiar in this case. Cf. Hudson, Kummer's Quartic Surface, Cambridge, 1905, page 196, and Zeuthen, Lehrbuch der abzählenden Methoden der Geometrie, Leipzig, 1914, page 276.

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## ON THE RECTIFIABILITY OF A TWISTED CUBIC.

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BY DR. MARY F. CURTIS.
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(Read before the American Mathematical Society December 31, 1919.)
If the space curve

$$
\begin{equation*}
x_{i}=a_{i} t^{n}+b_{i} t^{n-1}+\cdots+k_{i} t+l_{i} \quad(i=1,2,3) \tag{1}
\end{equation*}
$$

is a helix, it is algebraically rectifiable. For if it is a helix, it makes with a fixed direction a constant angle and $\sqrt{x^{\prime} \mid x^{\prime}}$ $=\left(x^{\prime} \mid \alpha\right)$,* where $\alpha_{1}, \alpha_{2}, \alpha_{3}$ are constants, not all zero; then the arc

$$
\begin{equation*}
s=\int_{t_{0}}^{t} \sqrt{x^{\prime} \mid x^{\prime}} d t \tag{2}
\end{equation*}
$$

is an integral rational function of $t$, not identically zero, and the curve (1) is algebraically rectifiable.

It is not, however, in general true, that if (1) is algebraically rectifiable, it is a helix. It will be true, provided (2) is an algebraic function only when $\left(x^{\prime} \mid x^{\prime}\right)$ is a perfect square of the form $\left(x^{\prime} \mid \alpha\right)^{2}$. This condition is fulfilled in the case of the twisted cubic:

$$
\begin{equation*}
x_{1}=a t, \quad x_{2}=b t^{2}, \quad x_{3}=c t^{3}, \quad a b c \neq 0 \tag{3}
\end{equation*}
$$

[^0]
[^0]:    * If $a:\left(a_{1}, a_{2}, a_{3}\right)$ and $b:\left(b_{1}, b_{2}, b_{3}\right)$ are two triples, then by $(a \mid b)$ we mean their inner product: $a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3}$.

