

with  $X_{n_2}Y_{n_2}$ . If this process is continued there will be obtained an infinite sequence of arcs  $X_{n_1}Y_{n_1}$ ,  $X_{n_2}Y_{n_2}$ ,  $X_{n_3}Y_{n_3}$ ,  $\dots$  no two of which have any point in common. For each  $i$ , the arc  $X_{n_i}Y_{n_i}$  contains, as a subset, an arc  $W_{n_i}Y_{n_i}$  which lies between the circles  $\bar{K}$  and  $K_1$ , except for the points  $W_{n_i}$  and  $Y_{n_i}$  which lie on  $K_1$  and  $\bar{K}$  respectively. There exist 1) on  $\bar{K}$  an infinite sequence of distinct points  $Y'$ ,  $Y'_1$ ,  $Y'_2$ ,  $Y'_3$ ,  $\dots$ , 2) on  $K_1$  an infinite sequence of distinct points  $W'$ ,  $W'_1$ ,  $W'_2$ ,  $W'_3$ ,  $\dots$ , 3) an infinite sequence of distinct arcs  $W'_1Y'_1$ ,  $W'_2Y'_2$ ,  $W'_3Y'_3$ ,  $\dots$  all belonging to the set  $W_{n_1}Y_{n_1}$ ,  $W_{n_2}Y_{n_2}$ ,  $W_{n_3}Y_{n_3}$ ,  $\dots$ , such that  $Y'$  is the sequential limit point of the sequence  $Y'_1$ ,  $Y'_2$ ,  $Y'_3$ ,  $\dots$  and  $W'$  is the sequential limit point of the sequence  $W'_1$ ,  $W'_2$ ,  $W'_3$ ,  $\dots$ . No two of the arcs  $W'_1Y'_1$ ,  $W'_2Y'_2$ ,  $W'_3Y'_3$ ,  $\dots$  have a point in common. It easily follows that there exists a closed connected point set  $N$ , containing  $Y'$  and  $W'$ , such that every point of  $N$  is a limit point of the point set constituted by the sum of the arcs  $W'_1Y'_1$ ,  $W'_2Y'_2$ ,  $W'_3Y'_3$ ,  $\dots$ . The point set  $N$  is a continuous set of condensation of the set  $M$ .

Thus the supposition that  $M$  is not connected "im kleinen" leads to a contradiction. It follows that  $M$  is a continuous curve.

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## DERIVATIVELESS CONTINUOUS FUNCTIONS.

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(Read before the American Mathematical Society October 26, 1918.)

THERE is no more interesting illustration of the refinement of geometric intuition through the influence of the arithmetization of mathematics than that presented by the history of functions of this type. No less a mathematician than Ampère, not to mention Duhamel and Bertrand, thought he had actually proved that continuous functions had derivatives for all save a finite number of arguments. Darboux in his paper on "Discontinuous functions" published in the *Annals* of the Ecole Normale for 1875, though dated January 20, 1874, in