analytic in (x_0, y_0, z_0) and r_1 and r_2 are not both zero there. These solutions of (3) are, then, analytic except perhaps in points of singularity of c_1, c_2 and in points for which $r_1 = r_2 = 0$. But they are identical with certain solutions of the equations (2), solved for $\partial k_1/\partial y$, $\partial k_2/\partial y$,—solutions which are analytic except perhaps in points of singularity of c_1, c_2 and in points for which $q_1 = q_2 = 0$. Evidently, then, these solutions k_1, k_2, and hence the vectors $\gamma_1 = k_1c_1 + k_2c_2$, $\gamma_2 = k_2c_1 - k_1c_2$, are analytic except perhaps in points of singularity of c_1, c_2 and in points in which both c_1 and c_2 are indeterminate, that is, have all three components zero. Thus we have the theorem:

If the gradients c_1, c_2 of the functions F_1, F_2 in $F = F_1 + iF_2$, where $F_1 = F_1 | [L] |$, $F_2 = F_2 | [L] |$ are functions of the first degree of the space curve L, are in general analytic, the gradients γ_1, γ_2 of Φ_1, Φ_2 in $\Phi = \Phi_1 + i\Phi_2$, an arbitrary complex function of L of the first degree isogenous to F, are analytic save perhaps in points of singularity of c_1 or c_2 and points in which both these vectors are indeterminate.

The theorem still holds when the vectors c_1, c_2 are proportional. In this case k_1 and k_2 are both solutions of the equation

$$p_i\frac{\partial k}{\partial x} + q_i\frac{\partial k}{\partial y} + r_i\frac{\partial k}{\partial z} = 0, \quad (i = 1, 2),$$

to which both of the equations (3) reduce in form, and γ_1 and γ_2 are both proportional to c_1 and c_2.

THE RICE INSTITUTE,
HOUSTON, TEXAS.

AN ELEMENTARY DERIVATION OF THE PROBABILITY FUNCTION.

BY CAPTAIN ALBERT A. BENNETT, C.A.R.C.

WE shall derive by means of elementary considerations the equation of the probability curve from the sequence of binomial coefficients. If the asymptotic form of $x!$ be obtained, the problem is very simple but none the less merits attention. The asymptotic form of $n!$, viz., $\sqrt{2\pi n}(n/e)^n e^{\theta/(12n)}$, $0 < \theta < 1$, might of course be taken for granted, but so far as is known