where

$$f(x) = r_0 + r_1 x + \dots + r_{p-2} x^{p-2},$$

$Z = e^{2i\pi/(p-1)}$, and r_i is the least positive residue of r^i, modulo p. Mr. Vandiver derives necessary and sufficient conditions that h be divisible by p^n, in terms of Bernoulli numbers, the argument used being different from those employed by Kummer and Kronecker in their treatment of the case $n = 1$.

20. In his first proof of the law of reciprocity between two ideals in a regular cyclotomic field Kummer gave the relation*

$$\left[\frac{d^{p-1} \log \omega (e^v)}{dv^{p-1}}\right]_{v=0} \equiv \frac{(\omega(1))^{p-1} - N(\omega)}{p} \pmod{p},$$

where

$$\omega(x) = a_0 + a_1 x + \dots + a_{p-2} x^{p-2},$$

$$\omega = a_0 + a_1 \alpha + \dots + a_{p-2} \alpha^{p-2},$$

$\alpha = e^{2i\pi/p}$ and $a_0, a_1, \dots, a_{p-2}$ are integers, ω being prime to the prime p. In the present paper Mr. Vandiver gives a proof of this result which differs in character from that of Kummer.

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NOTE ON ISOGENOUS COMPLEX FUNCTIONS OF CURVES.

BY PROFESSOR W. C. GRAUSTEIN.

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Let L be a continuous, closed and directed space curve, without multiple points, let $-L$ be the same curve oppositely directed, and let $F|[L]|$ be a function of L, such that $F|[-L]| = -F|[L]|$. Form the ratio

$$\frac{F|[L']| - F|[L]|}{\sigma},$$

where L' is the directed curve obtained from L by replacing an arc PP' of L by a second arc joining P with P', and σ

* *Abhandlungen, Berlin Academy,* 1859, p. 119, formula (7).