where, it is to be remembered, $\Delta^n w = \Delta^n w_0/5^{n+1}$ (and $w = w_0/5$). It should be noticed, however, that $\delta^4 u_1$ is expressed more simply by $.002\Delta^3 w_0$.

The arbitrary changes just suggested give an error of only about one half of one per cent of $\Delta^3 w_0$ in w_1 (the sum of the individual values interpolated). The error in each individual value would then be much less and if only two more decimal places are used than are to be finally retained—which are all that are ordinarily necessary as found by experience—the errors would not appear at all in the results. As a check upon the work the sum of the individual values interpolated should be w_1 as given originally.

It should be pointed out that the formulas derived above can not be used for "end" values; that is, if the groups of values were $w_0, w_1, w_2, \cdots, w_n$ the formulas could not be used for interpolating or breaking up w_0 or w_n, for the derivation of the formulas is based upon the use of four values (w_0, w_1, w_2, and w_3) to break up w_1; that is, there must always be at least one group preceding the group to be broken up. To break up "end" values formula (1) could be used or the formulas for the leading term and differences to be found in the article cited.

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NON-EUCLIDEAN GEOMETRY.

Geometrical Researches on the Theory of Parallels. By N. LOBACHEVSKI. Translated from the original by G. B. HALSTED. New edition. Chicago and London, Open Court, 1914. 8vo. 50 pages. Cloth, price \$1.25.

NON-EUCLIDEAN geometry had two independent discoverers: Johann Bolyai (1802–1860), a Hungarian officer in the Austrian army; and Nicolaus Lobachevsky (1793–1856), son of a Russian peasant, and graduate, professor, and rector, of the University of Kasan.*

As early as 1823 the former had grasped the real nature of his problem, and in 1829 he sent a completed manuscript on the subject to his father, Wolfgang Bolyai, who was a

* The best history of non-euclidean geometry is by Bonola. It was translated into English and edited by H. S. Carslaw (Chicago, 1912).