$$f_i^{(n)}(t),\ f_i^{(n+1)}(t),\ \cdots,\ f_i^{(n+q-1)}(t)$$

plus a sum of $q$-rowed determinants each of which has at least one column consisting of the derivatives of $f_1(t)$, $f_2(t)$, $\cdots, f_q(t)$ of an order less than $n$.

Using this fact we find upon differentiating the determinant $\Delta$ $(\lambda_1 + 1)(\lambda_2 + 1)$ times with respect to $t_2$ and putting $t_2 = t_1$ that the result is equal to a positive integer multiplied by the $p$-rowed determinant whose $i$th row is

$$f_i(t_1),\ f_i'(t_1),\ \cdots,\ f_i^{(\lambda_1)}(t_1),\ f_i^{(\lambda_1+1)}(t_1),\ \cdots,\ f_i^{(\lambda_1+\lambda_2+1)}(t_1),$$
$$f_i(t_3),\ \cdots,\ f_i^{(\lambda_3)}(t_3),\ \cdots,\ \cdots,\ f_i(t_\mu),\ \cdots,\ f_i^{(\lambda_\mu)}(t_\mu).$$

The other determinants which result from the differentiation drop out when $t_2$ is put equal to $t_1$, since each of them then has at least two columns identical. Repeating this process for the other variables in turn, we finally have the $p$-rowed determinant whose $i$th row is

$$f_i(t_1),\ f_i'(t_1),\ \cdots,\ f_i^{(p-1)}(t_1)$$

(or the wronskian of the $p$ given functions) vanishing identically if the determinant of the theorem does. The necessity of the condition of the theorem follows immediately as in the proof concerning the wronskian.

HARVARD UNIVERSITY,
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# ON THE LINEAR DEPENDENCE OF FUNCTIONS OF ONE VARIABLE.

BY DR. G. M. GREEN.

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As is well known, the identical vanishing of the wronskian of $p$ functions of a single variable is a sufficient condition for their linear dependence if the functions are analytic; if, however, they are not analytic the vanishing of the wronskian is not sufficient. The same situation arises in connection with the theorem proved by Mr. Morse and by Dr. Pfeiffer in the present number of the BULLETIN. The former makes explicit use of the analytic character of the functions involved, whereas the theorem proved by the latter may be neatly stated only for analytic functions, if it is to afford a criterion for linear dependence.