12. If $K_1(x, y)$ and $K_2(x, y)$ are functions of the form (1), then it is easily shown by means of (5) that the integral combination

$$\int_a^b K_1(x, \xi)K_2(\xi, y)d\xi \tag{16}$$

is of the same form, provided that the hypothesis of periodicity (the period being $b - a$) holds for the parts Ψ_1, Θ_1 and Ψ_2, Θ_2 of K_1 and K_2. This is the integral combination which has been so extensively studied by Volterra.

Now it is known that if we are given two functions K_1 and K_2 of x and y, and their respective resolvent kernels k_1 and k_2, there may be built up out of them by means of the combination (16) a new kernel and its resolvent; in fact, the function

$$k_1(x, y) + k_2(x, y) - \int_a^b k_1(x, \xi)k_2(\xi, y)d\xi \tag{17}$$

is resolvent for the function

$$K_2(x, y) + K_1(x, y) - \int_a^b K_2(x, \xi)K_1(\xi, y)d\xi.* \tag{18}$$

We have then the theorem:

If we have $K_1(x, y) = \Psi_1(x + y) + \Theta_1(x - y)$ *and* $K_2(x, y) = \Psi_2(x + y) + \Theta_2(x - y)$, *where* Ψ_1, Θ_1 *and* Ψ_2, Θ_2, *as functions of a single argument, are periodic with period* $b - a$, *and if we denote by* $k_1(x, y)$ *the function resolvent to* $K_1(x, y)$, *and by* $k_2(x, y)$ *the function resolvent to* $K_2(x, y)$, *then the functions given by* (17) *and* (18) *are of the same form, have the same properties of periodicity, and are themselves mutually resolvent kernels.*

RICE INSTITUTE,
April, 1916.

OPERATORS IN VECTOR ANALYSIS.

BY DR. VINCENT C. POOR.

IN a note in the April BULLETIN on "Changing surface to volume integrals," Professor E. B. Wilson asks why my paper in the January BULLETIN was not made shorter by using the Gibbs-Wilson notation. While the brevity and suggestiveness

* See the footnote to Section 10. For purposes of symmetry and convenience of statemen we have taken $\lambda = 1$ and assumed it not to be a root of K_1 or K_2.