1916.] 295ON PIERPONT'S DEFINITION OF INTEGRALS. then

$$F(x) = \int_0^\infty \varphi(t) t^{x-1} dt.$$

II. If $\varphi(x)$ satisfies the conditions in Theorem I, and if $F(t) = \int_0^\infty \varphi(x) x^{t-1} dx$, then reciprocally,

$$\varphi(x) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} F(t) x^{-t} dt.$$

Example:

$$\begin{split} \Gamma(x) &= \int_0^\infty e^{-t} t^{x-1} dt \,, \\ e^{-t} &= \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} \Gamma(x) t^{-x} dx \, \text{ for } a > 0; \; -\frac{\pi}{2} < \text{arg. } t < \frac{\pi}{2}. \end{split}$$

Miss Schottenfels' paper treats of a class of functions which are self-reciprocal in the above sense of reciprocity.

> H. E. SLAUGHT, Secretary of the Chicago Section.

ON PIERPONT'S DEFINITION OF INTEGRALS.

BY PROFESSOR M. FRÉCHET.

(Read before the American Mathematical Society, December 27, 1915.)

In the second volume of his Lectures on the Theory of Functions of Real Variables, Professor J. Pierpont has given a new definition of Lebesgue integrals. This definition is interesting in as much as it realizes an effort to adapt the previous methods of presentation of Riemann integrals to the newer Lebesgue integrals.

But unfortunately the happiness of this idea is lessened in Pierpont's work by the choice of an inappropriate definition. Professor Pierpont intended to generalize the definition of Lebesgue integrals by defining upper and lower integrals of any function f(x) on any linear set E_{μ} . Such definitions should not, of course, be arbitrary ones, and there are some primary conditions to be fulfilled, unless these definitions are to become quite artificial and uninteresting.