INVARIANTS, SEMINVARIANTS, AND COVARIANTS OF THE TERNARY AND QUATERNARY QUADRATIC FORM MODULO 2.

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1. A SIMPLE and complete theory of seminvariants of a binary form modulo p was given in the writer's second lecture at the Madison Colloquium.* A fundamental system of covariants of a ternary quadratic form F modulo 2 was obtained in the fourth lecture. In place of the method employed there (pages 77–79) to obtain the leading coefficient of a covariant of F, we shall now present a simpler method which makes it practicable to treat also the corresponding question for quaternary quadratic forms. The new method is, moreover, in closer accord with the underlying principle of those lectures, viz., to place the burden of the determination of the modular invariants upon the separation of the ground forms into classes of forms equivalent under linear transformation. By making the utmost use of this principle, we shall obtain a simpler solution of the problem for the ternary case and then treat the new quaternary case.

Let the coefficients of the quadratic form

$$q_n = \Sigma b_i x_i^2 + \Sigma c_{ij} x_i x_j \quad (i, j = 1, \cdots, n;\ j > i)$$

be undetermined integers taken modulo 2. In a covariant of order ω of q_n, the coefficient of x_n^ω is called the leader and also a seminvariant. It is invariant with respect to the group G generated by the linear transformations on $x_1, \cdots, x_{n-1}$ and those replacing x_n by $x_n + l$, where l is a linear function of $x_1, \cdots, x_{n-1}$, the coefficients in each case being integers taken modulo 2.

2. For $n = 2$, G is composed of the transformations

$$x_1 = x_1', \quad x_2 = x_2' + t x_1'.$$

Taking $t = b_1$ and applying the transformation to

* American Mathematical Society Colloquium Lectures, volume IV, New York, 1914; cited later as Lectures.