

SHORTER NOTICES.

Leçons sur le Calcul des Variations. Par J. HADAMARD.
Recueillies par M. FRÉCHET. Tome Premier. Paris, Hermann, 1910. 8vo. viii+520 pp. 18 francs.

A NEW aspect is given to the calculus of variations as it is presented by Hadamard in his emphasis upon the functional character of the whole problem. While this aspect of the subject has been mentioned by several writers before Hadamard's treatise appeared, it is an innovation in a formal treatment of the entire subject. The ideas of the functional calculus do not appear on the surface of things in a manner offensive to the traditions, but they are subconsciously present throughout the book, as is explained by Hadamard himself in the preface; and they crop out at the surface occasionally.

Briefly stated, any line integral between fixed end points, for example, is a function of the path of integration, in the sense that when that path is given, the integral is determined. From this standpoint, a problem of the calculus of variations is a problem in finding the maximum or minimum values of a functional expression, the independent variable being a *curve*. Of course, we wish also to determine the value of the independent variable—that is the curve—for which the extreme value occurs.

Since the book is now well known to all who are interested in the calculus of variations, this review must be a retrospect and an appreciation rather than an analysis. Much as the reviewer regrets the obstacles which have prevented an earlier review, this changed viewpoint is not without advantages.

Hadamard at once commanded respectful attention everywhere; an immediate review would not have increased nor diminished the desire for instant examination of the work. The reception the book has received throughout the world fully justifies this sweeping statement. Thus Carathéodory in the *Bulletin des Sciences Mathématiques* (volume 34, pages 124–142) calls it an epoch-making work (un livre qui marquera une date dans l'histoire du sujet). I will content myself with noting briefly the plan of organization and what appear to be the guiding motives.