for the extremal QP at its intersection with k_2. Let $J_{QP} = S(v)$; then

$$J_{64} = S(v_4), \quad J_{c_0} = J_{12} = S(v_2),$$

$$J_{64} - J_{c_0} = -[S(v_2) - S(v_4)] = -\int_{v_4}^{v_2} \frac{dS}{dv}\,dv$$

$$= -\int_{v_4}^{v_2} [x_2' H_{x'} + y_2' H_{y'}] dv,$$

and we have for the total variation

$$\Delta J = J_{\bar{c}} - J_{c_0} = \int_{\tau_3}^{\tau_4} E d\tau - \int_{v_4}^{v_2} [x_2' H_{x'} + y_2' H_{y'}] dv.$$

At the point 2, C_0 cuts k_2 transversally, that is,

$$x_2'(v) H_{x'} + y_2'(v) H_{y'} \big|^2 = 0,$$

and in this problem, as in the simple problem of the calculus of variations, we are led to a study of the sign of E along the extremal C_0 when considering sufficient conditions.

University of Illinois,
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A NOTE ON GRAPHICAL INTEGRATION OF A FUNCTION OF A COMPLEX VARIABLE.

BY DR. S. D. KILLAM.

(Read before the American Mathematical Society, April 26, 1913.)

The object of this paper is to give a shorter and purely graphical method for graphical integration than that of the author in his thesis* on graphical integration of functions of a complex variable.

We can represent a function $f(z)$ of the complex variable $z = re^{i\theta}$ in the $f(z)$-plane by a system of orthogonal curves $r = r_n$ $(n = 0, 1, \cdots, n)$ and $\theta = \theta_n$ $(n = 0, 1, \cdots, n)$. We choose the values r_n and θ_n so that the $f(z)$ plane is covered by a net of small squares. We seek now a graphical representation in the $Z = X + iY$-plane of the function $Z = \int_0^{z_n} f(z) dz$,

* "Über graphische Integration von Funktionen einer complexen Variabeln mit speziellen Anwendungen," Dissertation, Göttingen, 1912. Referred to in this paper as "thesis."