$(\mathfrak{M}_L)_* = \mathfrak{M}_*$ (cf. § 44a4). Also if $\mathfrak{M}_L$ has the properties D, Δ, so does $(M_L)_*$ (§ 79.2; § 44a5). In view of these propositions and Theorem II we have the following theorem.

THEOREM III. *If a class $\mathfrak{M}$ is composed of bounded functions μ and has the property D, then the necessary and sufficient condition that $\mathfrak{M}_L$ have the property Δ is that $\mathfrak{M}_*$ have the property Δ.*

In his dissertation, Chicago, 1912, E. W. Chittenden has made very effective use of infinite developments of a range $\mathfrak{P}$ where each stage of the development may contain a denumerably infinite number of subclasses. The theorems here given are valid also for such infinite developments. Theorem I may be established for infinite developments by essentially the same reasoning as above and in fact the same system $((\delta^{ml}))$ used above serves also in the case of infinite developments. The other theorems are established precisely as above.

DARTMOUTH COLLEGE,
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THE ASYMPTOTIC FORM OF THE FUNCTION $\Psi(x)$.

BY MR. K. P. WILLIAMS.

(Read before the American Mathematical Society, April 26, 1913.)

THE function

$$\Psi(x) = -C - \sum_{s=0}^{\infty}\left(\frac{1}{x+s} - \frac{1}{s+1}\right),$$

where C is Euler's constant, is of great importance in many questions in analysis, and also in certain problems in mathematical physics. It is the logarithmic derivative of the gamma function, and plays a fundamental rôle in the study of the latter. On account of the slow convergence of the series which defines it, the knowledge of the asymptotic form of $\Psi(x)$ is particularly desirable.* This can be computed directly from the above expression by the aid of factorial series,†

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* We use the term asymptotic according to the definition of Poincaré, and denote such a relation by the symbol $\sim$. See Borel, Les Séries divergentes, p. 26.

† Nielsen, Handbuch der Theorie der Gammafunktion, Kapitel XXI.