mations (1), (2), and (3) be denoted by $\varphi(z_3)$. The latter function is transformed by (4) into $\varphi(-z_3 + 2a)$. From the hypotheses of the theorem and the properties of the transformations employed, it follows that the function

$$\varphi(z_3) \cdot \varphi(-z_3 + 2a)$$

is analytic throughout the interior of σ and vanishes at every point of the boundary. Hence both the real and the pure imaginary parts of this function vanish at every point of the boundary of σ and are, therefore, both identically zero, since a function that is single-valued and harmonic throughout the interior of a region and vanishes at every point of the boundary is identically zero.* Since

$$\varphi(z_3) \cdot \varphi(-z_3 + 2a) \equiv 0,$$

one of the factors vanishes identically, and therefore

$$f(z) \equiv 0.$$

MATHEMATICAL PHYSICS AND INTEGRAL EQUATIONS.

Die Integralgleichungen und ihre Anwendungen in der mathematischen Physik. Vorlesungen an der Universität zu Breslau, gehalten von ADOLF KNESER. Braunschweig, Vieweg, 1911. 8vo. viii+243 pp.

THE solution of various boundary value problems for a partial differential equation by means of the expansion of an arbitrary function in series of solutions of ordinary differential equations involving a parameter constitutes one of the most important applications of the theory of integral equations. Here, as so often elsewhere, mathematical physics has first propounded the question, and it has been the task of analysis to furnish the answer. Especially close, therefore, has been the connection between mathematical physics and integral equations; especially interesting must be likewise a method of treatment which aims to exhibit this connection as vividly as possible. Such is the method of Kneser's book; we learn

* Osgood, Lehrbuch der Funktionentheorie, vol. 1, 2d ed., 1912, p. 623.