54. The varieties of self-projective quartic and quintic curves have been tabulated for the general case by Ciani and Snyder respectively. Dr. Winger in his paper presents the projectively distinct types of the most general rational curves of these orders which are invariant under the different finite collineation groups. The quartics are readily obtained from the consideration of the Stahl binary sextic. Six types are found with characteristic groups of orders 2, 3, 4, 4, 6 and 24, the first three being cyclic, besides one with an infinite group.

Of the quintics, two admit a one-parameter group. The others belong to cyclic groups of orders 2, 3, 4, 5 (two types), and dihedral groups of orders 4, 6, and 10 (two types),—eleven in all.

F. N. Cole,
Secretary.

THE PRODUCT OF TWO OR MORE GROUPS.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, December 31, 1912.)

§ 1. *Introduction.*

If H_1 and H_2 are any two groups, the symbol $H_1 \cdot H_2$ denotes the totality of the products obtained by multiplying each operator of H_1 on the right by every operator of H_2. A necessary and sufficient condition that this totality constitutes a group is that $H_1 \cdot H_2 \equiv H_2 \cdot H_1$. As $H_1 \cdot H_2$ is always composed of the inverses of all the operators represented by $H_2 \cdot H_1$, irrespective of whether this product is a group or does not have this property, we may also say that *a necessary and sufficient condition that* $H_1 \cdot H_2$ *is a group is that it includes the inverse of each one of its operators.*

Suppose that H_1 and H_2 have exactly h_0 operators in common. These common operators constitute a subgroup H_0, which is known as the cross-cut of H_1 and H_2. It is easy to prove that the number of the distinct operators in $H_1 \cdot H_2$ is always h_1h_2/h_0, where h_1 and h_2 represent the orders of H_1 and H_2 respectively. To see that $H_1 \cdot H_2$ cannot involve more than this number of distinct operators, it is only neces-