If the constants a_{pq} are chosen so that the determinants Δ_n are all positive, $D_{n-2}(\lambda)$ and $D_n(\lambda)$ will have opposite signs when $D_{n-1}(\lambda)$ vanishes, and so the functions

$$D(\lambda), \quad D_1(\lambda), \quad D_2(\lambda), \quad \cdots, \quad D_n(\lambda)$$

will form a Sturmian sequence.

It has been stated that the roots of the functions $\nabla_n(\lambda)$ in the Sturmian sequence separate one another. This is not always true for a Sturmian sequence when the functions are not polynomials, but it can be shown to be true in the present case, as follows. Let $g_n(s)$, $g_n(t)$ be the cofactors of the constituents $f_n(t)$, $f_n(s)$ in the determinant F_n; then from the properties of determinants

$$F_{n-1} \cdot \Delta_n - g_n(s)g_n(t) = F_n \cdot \Delta_{n-1}.$$

Dividing out by $\Delta_{n-1}\Delta_n$, we have

$$h_n(s, t) = h_{n-1}(s, t) - \frac{g_n(s)g_n(t)}{\Delta_{n-1}\Delta_n}.$$

We can now apply the theorem mentioned before to this equation and deduce that the roots of $h_{n-1}(s, t)$ are separated by those of $h_n(s, t)$, there being one root of $h_n(s, t)$ between each consecutive pair of roots of $h_{n-1}(s, t)$.

BRYN MAWR COLLEGE,
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ON THE CUBES OF DETERMINANTS OF THE SECOND, THIRD, AND HIGHER ORDERS.

BY PROFESSOR ROBERT E. MORITZ.

(Read before the San Francisco Section of the American Mathematical Society, April 8, 1911.)

WHILE the square of a determinant of any order may be readily expressed as a determinant of the same order, I am not aware of the existence of a correspondingly simple method by means of which the cube of any determinant may be expressed in determinant form. For a determinant of the fourth order, Δ_4, we have indeed from a well-known property of determinants

$$\Delta_4^3 \equiv \Delta_4',$$

where Δ_4' is the determinant whose constituents are the co-