A NOTE ON THE THEORY OF SUMMABLE INTEGRALS.

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§ 1. *Introduction.*

OF recent years increasing use has been made of infinite series which do not converge; but whereas the mathematicians before the time of Abel and Cauchy used such series without proper examination of the validity of their use, modern mathematicians in general employ them only when the legitimacy of the work can be clearly demonstrated. The need so arising for theories to justify the application of ordinary methods and transformations to non-convergent series just as if they did converge has been fully recognized, and to meet it there have been published many well-known memoirs by Cesàro, Borel, Poincaré, LeRoy, and others. The theory is still being rapidly extended, and new important applications are constantly appearing. Among recent recent workers on the subject may be mentioned Bohr, Bromwich, Fejer, Hardy, C. N. Moore, and Riesz.

Parallel with the theory for infinite series there is a theory for infinite integrals, but the latter has not yet been developed so much as the former. Mr. Hardy* seems to have been the first to define a "summable" integral, and his paper was closely followed by one due to Dr. C. N. Moore,† in which some properties of summable integrals were proved. Subsequently Dr. Bromwich ‡ wrote on the same subject, and more recently still the theory has been generalized by Mr. Hardy§ and

* *Quar. Jour. of Mathematics*, vol. 35, p. 54.

† "On the introduction of convergence factors into summable series and summable integrals," *Trans. Amer. Math. Society*, vol. 8, p. 299.

‡ " On the limits of certain infinite series and integrals," *Math. Annalen*, vol. 65, p. 350.

§ G. H. Hardy, "Notes on some points in the integral calculus," *Messenger of Mathematics*, vol. 40. "Theorems connected with Maclaurin's test for the convergence of series," *Proc. Lond. Math. Soc.* G. H. Hardy and S. Chapman, "A general view of the theory of summability of series and integrals," *Quar. Jour. of Mathematics*, 1911.