

Hence two contact transformations with the same transversality law will have a point transformation for alternant only when they are of the type

$$W = \sqrt{\alpha + 2\beta p + \gamma p^2}, \quad W_1 = \lambda \sqrt{\alpha + 2\beta p + \gamma p^2}.$$

Transversality is then expressed by a linear involutorial relation (15), so that for each point the transversal of a given direction is the conjugate direction with respect to a conic with that point as center.

8. A less important converse result, relating to the type considered in § 3, we state without proof. The only contact transformations which in combination with *every* transformation of type  $W = \Omega \sqrt{1 + p^2}$  give a point transformation for alternant are those of the same type. The same is true even if  $\Omega$  is restricted to the form  $a(x^2 + y^2) + bx + cy + d$ , a case of interest since then  $W$  converts circles into circles. When  $a$  vanishes the transformation belongs to the equilog class of Scheffers.

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## ON AN INTEGRAL EQUATION WITH AN ADJOINED CONDITION.

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IN his doctor dissertation \* Professor Cairns develops for infinitely many variables the theory of a quadratic form with an associated linear form, in order to prove the existence of solutions of the following integral equation :

$$(1) \quad \phi(s) = \lambda \int_a^b K(s, t) \phi(t) dt + \mu p(s),$$

with the adjoined condition

$$(2) \quad \int_a^b \phi(s) p(s) ds = 0,$$

where  $K(s, t)$  is a given continuous symmetric function of  $s$  and  $t$ ,  $p(s)$  a given continuous function of  $s$ ,  $\lambda$  and  $\mu$  are parameters, and  $\phi(s)$  is the function to be determined.

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\* "Die Anwendung der Integralgleichungen auf die zweite Variation bei isoperimetrischen Problemen," Göttingen, 1907.