THE INFINITESIMAL CONTACT TRANSFORMATIONS OF MECHANICS.

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1. THE significance of contact transformations in the development of general dynamics and optics, appreciated to some extent by Hamilton, was first brought out explicitly by Lie.* A very thorough and elegant discussion of the whole subject, including a number of new results, has recently been given by Vessiot.† With a conservative dynamical system, defined by its potential energy U (a function of n generalized coordinates) and its kinetic energy T (a quadratic form in the n generalized velocity components), there is associated an infinitesimal contact transformation whose characteristic function W is of special type.‡ The main result of the present note may be stated as follows:

The alternant (Klammerausdruck) of the contact transformations associated with two dynamical systems, of the same number of degrees of freedom, will be a point transformation when, and only when, the expressions for the kinetic energies are either the same or differ merely by a factor.

2. For simplicity and clearness we shall confine ourselves to two degrees of freedom. The infinitesimal contact transformations are then defined by a characteristic function $W(x, y, p)$, each lineal element (x, y, p) being converted into a neighboring element $(x + \delta x, y + \delta y, p + \delta p)$ according to the standard formulas

$$(1) \quad \delta x = W_p \delta t, \quad \delta y = (p W_p - W)\delta t, \quad \delta p = -(W_x + p W_y)\delta t.$$

If the transformation is applied repeatedly to any given element, a series of ∞^1 elements is obtained, the locus of whose points is termed a path curve or trajectory. The direction of the path generated by any element is defined by the formula

* "Die infinitesimalen Berührungstransformationen der Mechanik," *Leipziger Berichte* (1889), pp. 145–153. Lie-Scheffers, Berührungstransformationen, p. 102.

† *Bulletin de la Societé Mathématique de France*, vol. 34 (1906), pp. 230–269.

‡ The constant of total energy h is assumed to have a given value, so the discussion is connected with the theory of *natural families*.