the relatively prime pairs α, β and a, b. We shall prove this fact in the following paragraph.

Since G is abelian $(s_1s_2)^\alpha = (s_2s_1)^\beta = (s_1s_2)^\beta$, and hence $(s_1s_2)^{\alpha-\beta} = s_1^{\alpha-\beta}\, s_2^{\alpha-\beta} = 1$, or $s_1^{\alpha-\beta} = s_2^{\beta-\alpha}$. Combining this equation with $s_1^a = s_2^b$, there results $s_1^{a(\alpha-\beta)} = s_2^{a(\beta-\alpha)} = s_2^{b(\alpha-\beta)}$, and hence

$$s_2^{(a+b)(\beta-\alpha)} = 1 = s_1^{(a+b)(\alpha-\beta)}.$$

As the orders of s_1, s_2 are limited and these operators must be commutative, this proves that *only a finite number of groups can be generated by two operators which satisfy both of the equations*

$$(s_1s_2)^\alpha = (s_2s_1)^\beta \quad \text{and} \quad s_1^a = s_2^b,$$

where α, β and a, b represent two pairs of relatively prime numbers. For instance, when these numbers are 4, 5 and 2, 3 G is the group of order 5. That is, if s_1, s_2 satisfy both of the equations

$$(s_1s_2)^4 = (s_2s_1)^5, \; s_1^2 = s_2^3,$$

they must generate the group of order 5. This result establishes close contact between the present note and the paper "On groups which may be defined by two operators satisfying two conditions," *American Journal of Mathematics*, volume 31 (1909), page 167.

A NOTE ON IMAGINARY INTERSECTIONS.

BY PROFESSOR ELLERY W. DAVIS.

IN the plane let there be a conic C and a line L. Set up a system of coordinates such that L is the line infinity, its pole O with regard to C is the origin, the axes OX and OY are conjugate with regard to C, while X and Y are their intersections with L. Furthermore let $x = \pm 1$, $y = \pm 1$ be tangents to C through Y and X respectively. Then $x =$ a constant passes through Y, while $y =$ a constant passes through x. All these lines are to be determined by the fact that any four convergents form a harmonic set when the constants in the right member are a harmonic set of numbers. In brief, C, L, and the coordinates are projectively transformed from a circle $x^2 + y^2 = 1$, the line infinity, and a rectangular system whose origin is the center of the circle. The equation of any line in the transformed coordinates is precisely the same as that of which it is the projection in the rectangular coordinates.