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dimensions is obtained. A subgroup of G_{n^2-1} is obtained when the elements of M satisfy certain conditions, as *e. g.*, the wellknown conditions defining the orthogonal group. Professor Newson's fundamental theorem lays down the necessary and sufficient conditions which its elements of M must satisfy in order to have a subgroup of G_{n^2-1} . He defines a complete family of automorphic forms ϕ_i which are homogeneous and symmetric functions in from 1 to n sets of n variables each. His theorem is: A necessary and sufficient condition for the existence of a subgroup of G_{n^2-1} is that the elements of M satisfy a set of equations $\phi_i = l_i$ consisting of a complete family of automorphic forms in the elements of the rows or columns of M, each equated to the corresponding coefficient of the family.

Families of lower degrees define continuous subgroups of $G_{n^{2}-1}$; after a certain degree is reached the subgroups become discontinuous; above a certain other degree the conditions are satisfied only by the identical transformation. The paper will be published in the Kansas University Science Bulletin.

32. Mr. Schweitzer contrasted the formal properties of Bolzano's linear series with his exposition of the series of Vailati (the system ${}^{1}R_{1}$) and showed how to extend Bolzano's series to *n* dimensions (n = 1, 2, 3, ...) by considering simple modifications of the axioms of dimensionality and extension in his system ${}^{n}R_{n}$. Application of the author's *n*-dimensional open and closed chains is made.

F. N. COLE, Secretary.

THE GROUPS WHICH MAY BE GENERATED BY TWO OPERATORS s_1, s_2 SATISFYING THE EQUATION $(s_1s_2)^{\alpha} = (s_2s_1)^{\beta}, \alpha$ AND β BEING RELATIVELY PRIME.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, September 13, 1909.)

SINCE s_1s_2 and s_2s_1 are of the same order and α , β are relatively prime, it results that this common order is prime to both α and β . Hence s_1s_2 and s_2s_1 are generated by either $(s_1s_2)^{\alpha}$ or $(s_2s_1)^{\beta}$, and the cyclic group generated by s_1s_2 coincides with the one generated by s_2s_1 . A direct consequence of this is that the group generated by s_1s_2 is invariant under the entire group G