

whose form shows directly that an invariant must be independent of x , y , ρ , and therefore a function of σ only.

If u , x belong to a ray passing through the margin of the image and the axial point of the diaphragm, and v , y similarly to a ray passing through the center of the image and the edge of the diaphragm, a geometric construction shows that the expression $uy - vx$ is equivalent to the product of refractive index, lateral radius of image, and tangent of angular semi-aperture of central pencil. The invariant σ is therefore identical with the expression which occurs in the well-known equation pointed out in a special case by Lagrange, but first found as general by Helmholtz.* The result here shows that this is essentially the only relation expressible in terms of invariants which gives a general property of the paraxial transformation.

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Analytische Geometrie der Ebene. By C. RUNGE. Leipzig, B. G. Teubner, 1908. 198 pp.

SHALL we regard elementary analytic geometry, the analytic geometry we teach in a first course to freshmen or sophomores, as a body of doctrine with which it is useful for the student to become acquainted, or shall we rather regard it as an instrument with whose use he is to be made so familiar that he shall always be ready to employ it even in a quite new problem? This is a question which every teacher of analytic geometry and every writer of a text-book on the subject is called upon to face. Upon its answer the nature of the text-book written or selected for use will depend. Few persons, it is true, would go to such an extreme as to adopt without qualification either of the views above referred to. Those who regard it as their main object to inculcate a beautiful and important doctrine would deem it essential that the student gain the power of making the application of the general theorems learned to concrete cases, and the teacher who regards elementary analytic geometry primarily as a method whose use is to be taught would not neglect the opportunity of explaining

* Handbuch der physiologischen Optik, 1. Auflage, p. 50.