point not contained in the simplex $S(i+1)$ that determines it. But that one point is a point of the simplex $S(m+1)$ by the very definition of simplex. Therefore, if one begins to count with the first set and counts through the sets in order, the number of points in the set numbered $i+1$ that have not been counted in any previous set is ${}_{m+1}C_{i+1}$. It follows that the number of points in the simplex $S(m+1)$ is

$$\sum_{j=1}^{m} {}_{m+1}C_j = 2^{m+1} - 2,$$

which is one less than the number of points in the $m$-space determined by the $m+1$ vertices of the simplex.

From this theorem it follows that the $l+1$ vertices of a simplex of order $l$ determine uniquely another point, namely, the one point of the $l$-space determined by the simplex that is not also a point of the simplex. It is convenient to call this point the point complementary to the simplex. The triads, tetrads, pentads, etc. of the Steiner problem are found as follows: Every simplex $S(2)$ determines a triad consisting of its two vertices and the complementary point; every simplex $S(3)$ determines a tetrad consisting of its three vertices and the complementary point; and, in general, every simplex $S(l-1)$, $l \leqq k+2$, determines an *$l$-ad* consisting of the $l-1$ vertices and the complementary point. There are no *$l$-ads* for $l > k+2$.

When $n = 2^6 - 1 = 63$, it is possible to arrange the $n$ elements in triads, tetrads, pentads, hexads, and heptads. There is no arrangement of the 63 elements in $l$-ads for $l > 7$. This special case was involved in Steiner's investigation of the configuration of the 28 double tangents of a quartic curve * and led him to propose for solution the "Combinatorische Aufgabe" which I have called "The tactical problem of Steiner."

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# ON THE SO-CALLED GYROSTATIC EFFECT.

BY PROFESSOR ALEXANDER S. CHESSIN.

(Read before the American Mathematical Society, April 24, 1909.)

IN computing the resisting couple of gyrostats or the so-called "gyrostatic effect" it is customary to assume that it is equal to $C\lambda\omega \sin\theta$, where $C$, $\lambda$, $\omega$ and $\theta$ denote respectively the moment of inertia of the gyrostat about its geometrical axis, the angular

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* *Journal für die reine und angewandte Mathematik*, vol. 49, pp. 265–272.