of orthogonal functions, leads, if applied to the case $f = g$ to Bessel's identity

$$(6)\quad \int_R \left[ f - \sum_{p=1}^{n} \phi_p \int_R f\phi_p dR \right]^2 dR = \int_R f^2 dR - \sum_{p=1}^{n} \left[ \int_R f\phi_p dR \right]^2,$$

from which Bessel's inequality immediately follows.

5. Theorems analogous to those of the present note involving finite sums or infinite series in place of integrals may be proved in a similar manner.

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# ON THE TACTICAL PROBLEM OF STEINER.

BY PROFESSOR W. H. BUSSEY.

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THE study of tactical configurations known as triple systems had its origin in two problems proposed independently by J. Steiner * and T. P. Kirkman. † The Steiner problem, which is the more general and includes the other, is as follows:

For what values of $n$ is it possible to arrange $n$ elements in sets of three, called triads, so that every set of two elements is contained in one and only one triad? If $n$ is a number for which there is such an arrangement in triads, are there other arrangements that cannot be obtained from it by a mere permutation of the elements? When such an arrangement in triads has been made, is it possible to arrange the $n$ elements in sets of four, called tetrads, so that no triad is contained in a tetrad and so that every set of three that is not a triad is contained in one and only one tetrad? When such an arrangement in tetrads has been made, is it possible to arrange the $n$ elements in sets of five, called pentads, so that no triad or tetrad is contained in a pentad, and so that every set of four that is not a tetrad and does not contain a triad is contained in one and only one pentad? In general, when an arrangement in *k-ads* has been made, is it possible to arrange the $n$ elements in sets of $k+1$, called $(k+1)$-*ads* so that no *l-ad* $(l \leqq k)$ is contained in a $(k+1)$-*ad*, and so that every set of $k$ elements that is not a

* *Journal für die reine und angewandte Mathematik*, vol. 45, p. 181.

† *The Lady's and Gentleman's Diary* for 1850. For other references to the literature of Kirkman's fifteen school girls problem see Ball's Mathematical Recreations and Essays, 4th edition, page 121.