NECESSARY CONDITIONS THAT THREE OR MORE PARTIAL DIFFERENTIAL EQUATIONS OF THE SECOND ORDER SHALL HAVE COMMON SOLUTIONS.

BY PROFESSOR C. A. NOBLE.

(Read before the San Francisco Section of the American Mathematical Society, September 26, 1908.)

IN a paper inspired by Hilbert's lectures in 1900, Yoshiye (*Mathematische Annalen*, volume 57) considers, among others, the following problem in the calculus of variations: To find the necessary conditions that the integral

$$\int_{u_0}^{u_1} [\lambda(z' - px' - qy') + \mu(p' - rx' - sy') + \nu(q' - sx' - ty')]du$$

shall vanish independently of the path of integration, whereby the two equations

$$F(x, y, z, p, q, r, s, t) = 0, \quad G(x, y, z, p, q, r, s, t) = 0$$

shall be satisfied. λ, μ, ν are arbitrary functions of u; p, q, r, s, t have the usual signification, i. e., $p = \partial z/\partial x$, $q = \partial z/\partial y$, etc.; the accents denote differentiation with respect to u.

The conditions which result upon consideration of this problem are that the two equations

$$\nu^2 F_r + \mu\nu F_s + \mu^2 F_t = 0, \quad \nu^2 G_r + \mu\nu G_s + \mu^2 G_t = 0$$

shall have a common solution in $\mu : \nu$, i. e., that the determinant

$$\begin{vmatrix} F_r & F_s & F_t & 0 \\ G_r & G_s & G_t & 0 \\ 0 & F_r & F_s & F_t \\ 0 & G_r & G_s & G_t \end{vmatrix}$$

shall vanish; and furthermore, that for the value of $\mu : \nu$ which satisfies these two equations the relation

$$\mu[F_t(G_y) - G_t(F_y)] + \nu[F_r(G_x) - G_r(F_x)] = 0$$

shall be satisfied. F_x, F_y, etc., in the foregoing, denote par-