The norm of the number on the left is found to be p. It seems impracticable to determine whether or not p has actual prime factors in the field of 128th roots of 1, but this is very improbable, as the class number in that field is a multiple of 21,121.*

The use of complex numbers appears to be of no assistance in the problem of determining whether F_n is prime or composite.

AN EXTENSION OF CERTAIN INTEGRABILITY CONDITIONS.

BY PROFESSOR J. EDMUND WRIGHT.

SUPPOSE there are n functions $a_1, a_2, \cdots, a_n$ of n independent variables $x_1, x_2, \cdots, x_n$, satisfying the conditions

$$\frac{\partial a_p}{\partial x_q} - \frac{\partial a_q}{\partial x_p} = 0$$

for all values of p and q. It is well known that the functions a must all be first derivatives of a single function V. Similarly, if there are $\frac{1}{2}n(n+1)$ functions a_{pr} such that $a_{pr} = a_{qp}$, satisfying the relations

$$\frac{\partial a_{pq}}{\partial x_r} = \frac{\partial a_{pr}}{\partial x_q}$$

for all values of p, q, r, then the a's must be second derivatives of a single function.

The following question arises in connection with an application of the theory of invariants of quadratic differential forms:

Suppose there are $n(n+1)$ functions H_{pq}, K_{pq} such that $H_{pq} = H_{qp}$, $K_{pq} = K_{qp}$, satisfying the conditions

$$\frac{\partial}{\partial x_r}(H_{pq}) + K_{pq}\frac{\partial Y}{\partial x_r} = \frac{\partial}{\partial x_p}(H_{qr}) + K_{qr}\frac{\partial Y}{\partial x_p},$$

for all values of p, q, r; Y being a given function of the variables; what are the conditions on the functions H, K?

We first consider the case of $2n$ functions $a_1, a_2, \cdots, a_n$; $b_1, b_2, \cdots, b_n$, satisfying the conditions

* Reuschle, Tafeln, p. 461.