GENERAL ALGEBRAIC SOLUTIONS IN THE LOGIC OF CLASSES.

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(Read before the San Francisco Section of the American Mathematical Society, February 29, 1908.)

THE following treatment of the problem of inference in the logic of classes possesses some interest from its analogy to general solutions in ordinary algebra. The character of the general solutions here considered is most simply illustrated by what may be called the generalized problem of the syllogism, which may be stated as follows:

Let x, y, z be three class symbols, and let

$$f_1(x, y) = 0, \quad f_2(y, z) = 0,$$

be any two propositions involving x, y and y, z respectively; then it is required to deduce a proposition

$$f_3(x, z) = 0$$

involving x and z but not y.

The most general forms of the above propositions are (writing x' for $1 - x$, etc.)

$$(1) \qquad f_1(x, y) = l_1xy + l_2xy' + l_3x'y + l_4x'y' = 0,$$

$$(2) \qquad f_2(y, z) = m_1yz + m_2yz' + m_3y'z + m_4y'z' = 0,$$

$$(3) \qquad f_3(x, z) = n_1xz + n_2xz' + n_3x'z + n_4x'z' = 0,$$

in which l, m, n are numerical coefficients; and the non-vanishing of any coefficient (as m_2) implies the vanishing of the corresponding class term (yz'). The problem is to express the coefficients in (3) in terms of those in (1) and (2).

A solution is obtained in simple and symmetrical form by regarding (1), (2), and (3) as particular cases of the most general proposition involving x, y, z,

$$(4) \quad f(x, y, z) = axyz + bxyz' + cxy'z + dxy'z' + ex'yz + fx'yz' + gx'y'z + hx'y'z' = 0.$$

By Boole's rule of elimination

$$f_1(x, y) = f(x, y, 1)f(x, y, 0).$$