shall be irreducible in a domain R are that $(c_1^2 - 4c_2 + 8)^{1/2}$ *be irrational, and that either* $l = [(1 + \frac{1}{2}c_2)^2 - c_1^2]^{1/2}$ *be irrational or else l rational and* $[\frac{1}{2}c_2^2 - c_1^2 - 2 \pm (c_2 - 2)l]^{1/2}$ *both irrational.*

11. The only linear fractional transformations which replace a reciprocal equation by a reciprocal equation are

$$x' = \pm \frac{\alpha x + \beta}{\beta x + \alpha} \qquad (\alpha^2 \mp \beta^2). \tag{11}$$

Then y, given by (3), undergoes the transformation

$$y' = \pm \frac{(\alpha^2 + \beta^2)y + 4\alpha\beta}{\alpha\beta y + \alpha^2 + \beta^2}. \tag{12}$$

The transformation on $\frac{1}{2}y$ is the square of (11).

UNIVERSITY OF CHICAGO,
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A NEW GRAPHICAL METHOD FOR QUATERNIONS.

BY PROFESSOR JAMES BYRNIE SHAW.

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1. ANY quaternion q may be written in the form $q = (w + xi) + (y + zi)j$. For convenience let us represent numbers of the form $w + xi$ (practically equivalent to ordinary complex numbers save in their products by j) by Greek characters, so that q may be written

$$q = \alpha + \beta j,$$

where for any number β we have $\beta j = j\bar{\beta}$, $\bar{\beta}$ being the conjugate of β.

The tensor of q is then the square root of the sum of the squares of the moduli of α, β. Also the scalar of q is $\frac{1}{2}(\alpha + \bar{\alpha})$, that is, the real part of α.

2. The product of $q = \alpha + \beta j$ and $r = \gamma + \delta j$ is

$$qr = (\alpha\gamma - \beta\bar{\delta}) + (\alpha\delta + \beta\bar{\gamma})j,$$

and also we have

$$rq = (\alpha\gamma - \bar{\beta}\delta) + (\bar{\alpha}\delta + \beta\gamma)j.$$