$$(5)\quad \begin{aligned} &D^2 = 1 + (y - e^{-1/x^2})(y - 2e^{-1/x^2}), \quad \text{when } x \neq 0\,; \\ &D^2 = 1 + y^2, \quad \text{when } x = 0\,; \end{aligned}$$

which is precisely the function

$$(6)\quad \begin{aligned} &\phi(x, y) = (y - e^{-1/x^2})(y - 2e^{-2/x^2}), \quad \text{when } x \neq 0\,; \\ &\phi(x, y) = y^2, \quad \text{when } x = 0\,; \end{aligned}$$

increased by unity; the function $\phi(x, y)$ given by (6) has been studied *; it is at a minimum for any analytic curve in the (x, y) plane through the point (0, 0); but it is not at a minimum in the region about (0, 0). Thus even the knowledge that the distance from a point on the normal to a surface is at a minimum at the foot of the normal for every curve cut out of the surface by an analytic cylinder through the normal, does not prove that the same distance is at a minimum for the surface itself.

COLUMBIA, MO.,
January 11, 1908.

A GEOMETRIC REPRESENTATION OF THE GALOIS FIELD.

BY DR. L. I. NEIKIRK.

(Read before the Chicago Section of the American Mathematical Society, March 30, 1907.)

THE roots of irreducible congruences were first introduced into mathematics by Galois.† Various writers since then have contributed to the theory of irreducible congruences and classes of residues.‡ The greatest progress of recent years was made by Moore.§ He proved that any finite abstract field in which division is unique is an abstract form of the Galois field; that its order is the power of a prime p^n, and that for any order it is unique, being independent of the particular irreducible congruence of degree n used in defining it.

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* *Annals of Mathematics*, vol. 8, no. 4 (July, 1907), pp. 172–174. The exact significance of "analytic," as here used, is there specified, and a more general statement of the property quoted is given.

† "Sur la théorie des nombres," *Bulletin des Sciences Math.* de M. Ferrussac (1830); also Œuvres mathématiques d'Evariste Galois, Gauthier-Villars, Paris, 1897.

‡ See the preface of Linear Groups by L. E. Dickson.

§ BULLETIN, December, 1893; Chicago Congress Mathematical Papers, pp. 208–242.