

## SUBJECTIVE GEOMETRY.

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THE fact that many persons suppose that the non-euclidean and euclidean geometries contradict each other seems to call for further elucidation of the matter. Things that disagree do not necessarily contradict each other. One circumstance that leads to the prevalent obscurity in this matter is the use of diagrams and objective phraseology in the treatment of synthetic geometry. The reader always supposes that geometric figures are something he can see and handle. Also, in Euclid's mode of presenting the science, it is difficult to see what is fundamental and what derivative.

It is said that Legendre cast about for a long time to find the proof of the theorem "The sum of the three angles of a triangle is equivalent to two right angles" without success. He should have seen that his failure arose from the want of due determination in his definition. The theorem is true if the proper concepts are attached to the words "triangle," "angle" and "right angle."

Let us illustrate by an instance from algebra. The man who knows nothing about negative and complex quantity will say "No cubic equation can have more than three roots, but there are some which have less." The man who admits these sorts of quantity says "Every cubic equation has three roots." These statements disagree, but are not in contradiction. The case is the same as if they were written in different languages. We can translate the first statement so as to agree with the second by simply writing "positive real root" instead of "root."

The difference between the non-euclidean and the euclidean geometry is one of mere definition of terms. We shall see this more easily if we treat geometry as a subjective science. There are two methods for its exposition called synthetic and analytic. The latter makes use of the symbolic notation of algebra, and is more easily comprehended if the notion of a linear unit is admitted. At the end of the investigation, however, the latter