$$G_{22} = \begin{vmatrix} \pm 1 & 0 & 0 & \cdots \\ 0 & \pm 1 & 0 & \cdots \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \pm 1 \end{vmatrix}, \quad G_{23} = \begin{vmatrix} 0 & 0 & \cdots & m_1 & n_1 \\ 0 & 0 & \cdots & m_2 & n_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & m_r & n_r \end{vmatrix},$$

$$G_{33} = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix},$$

in which $m_i\ n_i\ \alpha,\ \beta,\ \gamma,\ \delta$ *are integers and* $\alpha\delta - \beta\gamma = 1$.

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TWO TETRAEDRON THEOREMS.

BY PROFESSOR HENRY S. WHITE.

THE sphere and the tetraedron yield two combinations familiar to students of geometry, those in which one object is inscribed in the other; and one less well known, that in which the edges of the tetraedron are tangents to the sphere. A novel theorem upon the circumscribed tetraedron was propounded in 1897 by Bang and proved by Gehrke, and has been made the starting-point for extended developments by Franz Meyer * (1903) and Neuberg (1907).† It is this: If the contact point in each face of a tetraedron circumscribed about a sphere be joined by a straight line to each vertex in its face, then three angles at each contact point are equal respectively to the three formed at any other contact point. Or it may be stated thus: Opposite edges of a circumscribed tetraedron subtend equal angles at the points of contact of the faces which contain them.

While elementary proofs of this are interesting, a more elaborate deduction is of value here as suggesting a second theorem. It can be made to depend upon the well-known theorem from the projective geometry of a straight line, namely,

* W. Franz Meyer: "Ueber Verallgemeinerungen von Sätzen über die Kugel und das ein- resp. umbeschriebene Tetraeder." *Jahresbericht der deutschen Mathematiker-Vereinigung*, 1903, p. 137.

† J. Neuberg: "Ueber die Berührungskugeln eines Tetraeders," *Jahresbericht der deutschen Mathematiker-Vereinigung*, 1907, p. 345.