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tically. This may be seen by substituting the particular values  $a_{11} = b_{22} = c_{33} = d_{44} = 1$ , the remaining  $a_{ij}$ , etc., being zero. Hence these quantities  $F_{ijkl}$  are independent.

Finally, consider the quartic

$$\theta_{2}(A, B, C, D, E) = 0.$$

Let this equation be reduced to

$$A^2 + B^2 + C^2 + D^2 + E^2 = 0.$$

Equate the coefficients of the terms of this equation to the corresponding terms of

$$\Sigma k_{ijkl} x_i x_j x_k x_l = 0 \qquad (i \le j \le k \le l \le 4)$$

and determine the jacobian matrix as before. That the determinants of this matrix do not all vanish identically is seen by taking for A, B, etc., the particular expressions

$$A \equiv x_1^2$$
,  $B \equiv x_2^2$ ,  $C \equiv x_3^2$ ,  $D \equiv x_4^2$ ,  $E \equiv x_1 x_2$ .

Since these determinants do not vanish identically the  $k_{ijkl}$  are independent. Hence the equation of an arbitrary quartic surface can be put into the form

$$A^2 + B^2 + C^2 + D^2 + E^2 = 0.$$

URBANA, ILL., September, 1907.

## SYMBOLIC LOGIC.

- L'Algèbre de la Logique. Par LOUIS COUTURAT. Collection Scientia, No. 24. Gauthier-Villars, Paris, 1905. 100 pp.
- Symbolic Logic and its Applications. By HUGH MACCOLL. Longmans, Green, and Co., London, 1906. xi + 141 pp.
- The Development of Symbolic Logic; a Critical-Historical Study of the Logical Calculus. By A. T. SHEARMAN. Williams and Norgate, London, 1906. xi + 242 pp.

SYMBOLIC logic is in the interesting though somewhat precarious state of being little known, less used, and much scorned by the majority of mathematicians and philosophers, for whom it might supposedly offer a region of intimate contact and

1908.]