ON THE SHORTEST DISTANCE BETWEEN CONSECUTIVE STRAIGHT LINES.

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CERTAIN well-known geometric results concerning space curves and surfaces have been obtained by a discussion of the shortest distance between consecutive positions of a straight line moving continuously in space. These results have been gained by a discussion (recapitulated in § 1) of the *numerator* of the expression for the shortest distance.* It is the purpose of this paper to complete the discussion by examining (§§ 2–3) the *denominator* of the distance expression, placing special emphasis upon the conditions that the distance be an infinitesimal of the *second* order, and upon a geometric interpretation of this case.

§ 1. *Brief Discussion of the Numerator.*

The equations of the straight line are

$$(1) \qquad \begin{aligned} x &= az + p, \\ y &= bz + q, \end{aligned} \qquad \text{or} \qquad \begin{aligned} x &= a(t)z + p(t), \\ y &= b(t)z + q(t), \end{aligned}$$

where a, b, p, q are analytic functions of a single variable t. We define the consecutive line by the equations

$$(2) \qquad \begin{aligned} x &= a(t + dt)z + p(t + dt), \\ y &= b(t + dt)z + q(t + dt), \end{aligned}$$

or

$$x = \left(a + a'dt + a''\frac{dt^2}{2\,!} + \cdots\right)z + \left(p + p'dt + p''\frac{dt^2}{2\,!} + \cdots\right),$$

$$y = \left(b + b'dt + b''\frac{dt^2}{2\,!} + \cdots\right)z + \left(q + q'dt + q''\frac{dt^2}{2\,!} + \cdots\right),$$

where $a' = da/dt, a'' = d^2a/dt^2, \cdots$. The formula for the shortest distance between lines (1) and (2) is given by †

* Koenigs: Géométrie réglée: *Annales de la Faculté des Sciences de Toulouse*, vol. 6, pp. 38–40, 61–63.
Joachimsthal: Anwendungen der Diff. und Int. Rechnung, etc., pp. 182–184.
Knoblauch: Einleitung in die allgemeine Theorie der krummen Flächen, pp. 104–106.

† Laurent: Traité d'analyse, vol. 2, pp. 298–303.