MODULAR THEORY OF GROUP CHARACTERS.

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1. THE problem of the representation of a given finite group as a linear homogeneous group with real or complex coefficients has been fully treated by Frobenius * by means of his theory of group characters. The present paper and the companion paper to appear simultaneously in the *Transactions* give a first attack on the corresponding problem for linear congruence groups, and in general for finite linear groups in any field F having a prime modulus p. To obtain simple results, it is in general necessary to introduce certain irrationalities, viz., roots of equations with coefficients in F. As our reference field we shall take the field F_p composed of the totality of integral rational functions with integral coefficients of all Galois imaginaries of all degrees, *i. e.*, the roots of congruences irreducible modulo p. In other words, F_p is the aggregate of the Galois fields $GF[p^n]$, $n = 1, 2, 3, \cdots$. Hence every equation with coefficients in F_p is completely solvable in F_p.

The paper also gives a report on the various expositions of the algebraic theory from the standpoint of their availability in the treatment of the modular theory (cf. §§ 3, 5, but particularly § 13).

2. *Definitions.* Given a finite group H with the h elements $H_0, H_1, \cdots, H_{h-1}$, we shall say that the h matrices of degree f (or linear substitutions)

$$A_{H_i} = (a_{\alpha\beta}^{H_i})_{\alpha,\beta=1,\cdots,f} \quad (i = 0, 1, \cdots, h-1), \tag{1}$$

whose elements a are marks of the field F_p, define a representation of the group H if the matrices satisfy the h^2 relations

$$A_R A_S = A_{RS} \quad (R, S = H_0, \cdots, H_{h-1}). \tag{2}$$

The matrices need not be distinct, so that the isomorphism may be multiple. Let $\mathrm{x}_{H_i} (i = 0, \cdots, h-1)$ be independent variables. Then

$$X = \sum_R A_R \mathrm{x}_R \quad (R = H_0, \cdots, H_{h-1}) \tag{3}$$

is called the group matrix corresponding to the representation.

* *Berliner Sitzungsberichte*, from 1896 to date.