ON A FINAL FORM OF THE THEOREM OF UNIFORM CONTINUITY.

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1. It is one of the fundamental theorems of analysis that a function which is continuous in a closed interval is uniformly continuous in that interval. A natural generalization to any closed assemblage is plausible and has been proved by Jordan;* again, an extension to a function of any number of variables is almost obvious. Another and a less obvious generalization has occurred to the writer, of which a special case has been stated by Baire.† In the case of a function of one variable this may be stated as follows:

Let $f(x)$ be a function defined on any assemblage (H) of values of x, and let (E) be a closed subassemblage of (H');‡ if the oscillation $\Omega(x)$ is defined with respect to the values of $f(x)$ on (H) in the usual manner§ and if $\Omega(x) \leqq k$ at each point of (E), then, corresponding to any positive number ϵ, there exists another positive number η, such that $|f(x) - f(\xi)| < k + \epsilon$ when ξ is any point of (E) and x is any point of (H) for which $|x - \xi| < \eta$.

The nature of the extension will appear directly; at this point attention is directed to the final character of this result, in its application to any function for values assumed on any assemblage.

2. The ordinary statements may be revised by use of the concept of oscillation, and it is desirable for what follows to do this. If a function $f(x)$ is defined on any assemblage (H) whose limiting points form another assemblage (H'), the values

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*Jordan, Cours d'analyse, 2d ed., vol. 1, p. 48.

† Baire, Thesis: "Sur les fonctions de variables réelles," *Annali di Matem.*, 1899, p. 15; Baire, Leçons sur les fonctions discontinues, Paris, 1905; Borel, Leçons sur les fonctions de variables réelles, Paris, 1905, p. 27. Baire's statement makes (H) a continuum and $(E) = (H)$. See also W. H. Young, Theory of sets of points, Cambridge, 1906, p. 218.

‡ (H') is the assemblage of all the limiting points of (H), *i. e.*, the *first derived assemblage.* It is not necessary that (E) be part of (H'), but the real content of the theorem is the same if this restriction is made.

§ See § 2.