we obtain surfaces of translation, applicable to one another and depending upon seven arbitrary parameters.

The conditions that there exist three constants $a, b, c$ such that for all values of $V$ and $V_{1}$ the curves $u=$ const. lie in the planes $a X+b Y+c Z=d$ are

$$
\begin{gathered}
a k-b-c g=0 \\
a\left(h b_{3}-k b_{2}\right)+b\left(k b_{1}-g b_{3}\right)+c\left(g b_{2}-h b_{1}\right)=0 \\
a\left[h+\left(h c_{3}-k c_{2}\right) e^{-a}\right]+b\left[-g+e^{-a}\left(k c_{1}-g c_{3}\right)\right] \\
+c\left[1+e^{-a}\left(g c_{2}-h c_{1}\right)\right]=0 .
\end{gathered}
$$

Equating to zero the determinant of these equations, we get in consequence of (19) the following equations of conditions

$$
\begin{align*}
&\left(k^{2}+h^{2}\right) b_{1}-(k+g h) b_{2}+(h-k g) b_{3}=  \tag{22}\\
& h\left(g a_{1}+h a_{2}+k a_{3}\right) e^{-a}
\end{align*}
$$

and the planes of the curve are parallel to the plane

$$
\begin{aligned}
& {\left[(h+g k) b_{1}-g b_{2}-g^{2} b_{3}\right] x+h\left(k b_{1}-g b_{3}\right) Y+} \\
& {\left[k^{2} b_{1}-k b_{2}+(h-k g) b_{3}\right] Z=0 .}
\end{aligned}
$$

As in the particular case previously considered, equation (22) is the condition also that the curves $v=$ const. on $S_{1}^{\prime}$ lie in parallel planes.

Another seven parameter aggregate of pairs of applicable surfaces of translation is found when the values from (1) and (2) are substituted in equations (20) and (21).

Princeton University, February, 1905.

## THE GROUPS OF ORDER $2^{m}$ WHICH CONTAIN AN INVARIANT CYCLIC SUBGROUP OF ORDER $2^{m-2}$.

 BY PROFESSOR G. A. MILLER.Hallet * has recently called attention to the fact that Burnside omits one group in his enumeration of the non-abelian groups of order $2^{m}$ which contain an invariant cyclic subgroup

[^0]
[^0]:    * Bulletin, vol. 11 (1905), p. 318 ; Science, vol. 21 (1905), p. 176.

