

let t represent any operator of order p which transforms the generator s of a cyclic group of order p^{a_1+1} into its $p^{a_1} + 1$ power. The commutator quotient group of the group generated by s and t is clearly of type $(\alpha_1, 1)$. Let t' represent any operator of order p^{a_2} which is independent of s and t . The operators s and tt' will then generate the required non-abelian group. By forming the direct product of this non-abelian group and some abelian group any additional invariants may be introduced into the commutator quotient group. Hence the theorem is proved.

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NOTE ON ISOTHERMAL CURVES AND ONE-PARAMETER GROUPS OF CONFORMAL TRANSFORMATIONS IN THE PLANE.

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IN the January number of the BULLETIN, page 180, Mr. J. E. Wright integrated a certain differential equation by determining a continuous group which the equation admits. In solving the problem Mr. Wright determines a group of *conformal* transformations with *given* path curves. In this connection, it is an obvious problem to find the necessary and sufficient conditions under which a conformal group with given path curves shall exist. The solution of this problem is given in the following theorem:

THEOREM.—*A one-parameter group of conformal transformations with given path curves exists when and only when the given curves form an isothermal family.*

Although this theorem seems very obvious the writer cannot find it in print, and, therefore, gives two easy proofs for it.

I. Let $Uf = \xi p + \eta q$ be the symbol of the infinitesimal transformation of the group. Since the group is to be conformal, we must have $\xi + i\eta = \phi(x + iy)$. The differential equation of the path curves is $\eta dx - \xi dy = 0$. From this equation we have

$$\frac{dx + idy}{\xi + i\eta} = \frac{dx - idy}{\xi - i\eta},$$