

Elements of the Theory of Integers. BY JOSEPH BOWDEN, Ph.D. New York, The Macmillan Company, 1903. x + 258 pp. Price \$1.25.

PROFESSOR BOWDEN has undertaken in this volume to base the elementary theory of numbers on the three fundamental ideas of number, equality and sum using the following axioms only :

1. $a = a$.
2. If $a = b$ then $b = a$.
3. If $a = b$ and $b = c$ then $a = c$.
4. $a + b = b + a$.
5. If $a = b$ then $a \neq c + b$.
6. If $a \neq b$ there is some number c such that either $a = c + b$ or $c + a = b$.
7. If $a = b$ then $a + c = b + c$.
8. If $a \neq b$ then $a + c \neq b + c$.
9. $(a + b)c = a + (b + c)$.
10. Every number except unity is the sum of two numbers.

This list the author submits as a set of independent and sufficient axioms.

The book is divided into two parts to which the author hopes to add three more. The first part deals with the fundamental ideas, axioms etc., together with the statement of certain principles of logic to be used in developing the subject. The notion of subtraction is introduced together with the ideas "greater than," "less than" etc. In this part the difference $a - b$ is not considered as having a meaning unless $a > b$. Numbers in this first part are distinguished as *primary* numbers. In the second part an integer is defined as a "number couple" made by combining two primary numbers a and b , this number couple being the difference $a - b$, when $a > b$ and in all cases having properties like those of a difference. The fundamental operations of addition, subtraction, multiplication and division are defined for these number couples.

There is a chapter on factors in which the sieve of Eratosthenes is described, and the resolution of a composite number into its prime factors is discussed. Chapters are also devoted to the greatest common factor and to the least common multiple. The book closes with a chapter on congruences.

Professor Bowden makes use of a very elaborate and unusual system of notation. The greatest common factor of α and β