ON THE CONDITION THAT A POINT TRANSFORMATION OF THE PLANE BE A PROJECTIVE TRANSFORMATION.

BY MR. ELIJAH SWIFT.

(Read before the American Mathematical Society, October 31, 1903.)

1. WHILE it is well known that all projective transformations are collineations, the converse has, I believe, never been proved in all its generality. Möbius, in his "Barycentrischer Calcul" proves by means of his net of lines that if we start from any four independent * fixed points in the plane, we can either reach any point of the plane, by means of constructions with a ruler alone, or else come as near it as we please. Thence he infers that there cannot be more than one collineation which carries four given independent points into four other given positions. Since we know that there exists a projective transformation which carries over four arbitrarily given independent points into four other arbitrarily given independent points, we infer that every collineation is a projective transformation. In this reasoning, however, Möbius clearly assumes that he is dealing only with transformations which are in general one-to-one and continuous. There are, however, points in the two planes (the points on the vanishing lines) where the transformation is, strictly speaking, not defined. Thus two questions present themselves:

(1) Is it necessary to require that the collineation be continuous in order that Möbius's theorem be true?

(2) Throughout what part of the plane may we leave the transformation undefined?

It is my object in this paper to prove the following theorem, which answers the first question, and goes a long way toward answering the second.

Suppose we have a one-to-one correspondence between the points of two plane point sets S and S', each of which has an interior point, such that any three collinear points in either set have collinear images. Then the transformation of S into S' is a projective transformation.

* By four independent points in a plane we understand four points no three of which lie on a straight line.