NOTE ON ISOTROPIC CONGRUENCES.

BY DR. L. P. EISENHART.

(Read before the American Mathematical Society, December 28, 1901.)

CONSIDER a sphere S of radius unity and center at the origin of coördinates, and a surface S_1 corresponding to S by orthogonality of linear elements. By a theorem of Ribaucour * we know that S_1 is the mean surface of an isotropic congruence C. If S_1 is taken to define an infinitesimal deformation of S, the associate† surface in this deformation will be a minimal surface S_2 ;‡ then S and S_2 correspond by parallelism of tangent planes at corresponding points. Moreover, if the reciprocal character of the relation existing between these latter two surfaces is noted, and S is considered as the associate in the deformation of S_2, then the surface S_3 corresponding to S_2 with orthogonality of linear elements is the adjoint minimal surface of S_2.

Darboux has shown § that the following relations exist between the cartesian coördinates of these four surfaces :

$$
\begin{aligned}
x_1 &= x_3 - y_2 z + z_2 y, \\
y_1 &= y_3 - z_2 x + x_2 z, \qquad (1)\\
z_1 &= z_3 - x_2 y + y_2 x.
\end{aligned}
$$

Let S be referred to its asymptotic lines ; then S_2 and S_3 will be referred to the double system of lines which is conjugate for each || As the latter are adjoint minimal surfaces, this double system is made up of the lines of length zero on each surface. Weierstrass has shown ** that the coördinates of S_3 can be expressed as the following functions of u and v, parameters referring to the lines of length zero :

$$
\begin{aligned}
x_3 = \frac{1-u^2}{2} f''(u) + uf'(u) - f(u) \\
+ \frac{1-v^2}{2} f_1''(v) + vf_1'(v) - f_1(v),
\end{aligned}
$$

* Étude des Elassoïdes, chap. 8 (Mémoire couronné par l'Academie de Belgique) ; Darboux, vol. 4, p. 15.
† Bianchi, p. 279.
‡ Darboux, vol. 4, p. 96.
§ Ibid., p. 66.
|| Bianchi, p. 284.
** Darboux, vol. 1, p. 289.