PASCAL'S CALCULUS OF VARIATIONS.

Die Variationsrechnung. Von E. PASCAL. Deutsch von A. SCHEPP. B. G. Teubner, Leipzig, 1899. vi. + 146 pp.

THE German translation of the "Calcolo delle variazioni" published by Ernesto Pascal in 1897 gives to American mathematicians in convenient form the best book on the calculus of variations that has, to our knowledge, appeared up to the present time. The book consists of only 150 octavo pages and presents concisely the principal facts of the subject. A valuable feature of the work will certainly be found to be the very excellent and apparently complete bibliography given in connection with brief accounts of the development of the calculus of variations. No book with which we are acquainted could be better adapted to controvert the lay opinion that everything in mathematics is exact and beyond dispute. Again and again the author calls attention to errors made by writers in this field. He often calls attention, too, to gaps which remain still to be filled in the theory, and makes the reader sometimes feel that the results which we already have rest on a rather precarious foundation. The chief fault of the book, from our point of view, is that it sacrifices simple and natural discussion to the pursuit of the end so dear to Italian mathematicians, the greatest possible generality.

The apparent purpose of the author is to give an account, absolutely rigorous as far as it goes, of the present condition of the science. That such an end is in the calculus of variations especially difficult to attain appears from the fact that the proofs are not always precise and that the author prefers often to tell us that the work given is not rigorous rather than to attempt to make it so. A serious deficiency is the almost entire lack of reference to the work of Weierstrass. This lack is, of course, intentional and doubtless due to the author's courtesy in hesitating to refer to what is to appear in Schwarz's edition of Weierstrass's works. It seems to us, however, that Pascal, with entire justice to Weierstrass, might have absorbed a little more of Weierstrass's spirit, and distinguished more carefully among the different sorts of variations employed, laying a proper emphasis on the function of each kind. The most unsatisfactory part of the book is the treatment of Jacobi's criterium. This condition is abstruse and difficult to grasp in all its beauty and ingenuity even in the simplest case. Pascal gives a purely analytical treatment of the general case. We ven-